INTRODUCTION

This paper is concerned with the utility of L.P. techniques in order to approximate a known multivariable function, specified by the total number of sample points, S, with a multivariable rational function. If $x_i \in \mathbb{R}^m$, $i = 1, 2, \ldots, s$, is the vector representing the function variables at a given point, then an approximation based on a minimum criterion analytic examples are presented. The proposed approach is efficient L.P. method requiring minimum memory storage. The approach is introduced in a new last and extensive memory requirement. This paper is introducing a new last and extensive memory requirement. Moreover, it encounters difficulties that are concerned with long computation times.

ABSTRACT

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On the Optimum Approximation of Real Rational Functions via Linear Programming
subject to the constraints

with

problem, with 3k constraints, consisting of

After the kth iteration, an approximation to (c)^x, is achieved, denoted by

for every \( x \in S \).

with the additional constraints

coefficients \( p_i \) and \( q_i \) by minimizing the quantity

an iterative procedure to achieve the optimum solution, i.e., to estimate the
functions. The DCA approach is based on a minimum formulation and follows
most widely accepted methodologies for the approximation of real rational
an exact solution, while Lee and Roberts [5] consider that it is precisely the
a global solution, while Lee and Roberts [5] and also by other authors [6]. The latter investigators
and later extended by BaradSTITUTE et al. [7]. The latter investigators
introduced DCA, originally introduced by Chan and
differentiation techniques. The most significant contribution is the

during the past years, several combinatorial problems like this problem have been

If \( \mathcal{C}(x) = (x)^x \) is the ideal function to be approximated in S, we seek to

equal to unity. If the polynomials \( (x)^x \) and \( (x)^d \) are all
then the polynomial rational function may be written as

where \( N \) are given integers, \( p_i \) and \( q_i \) are coefficients of \( x \) whose coefficients are all

where only \( b' \) and \( d' \) are integers, \( N' \) and \( N' \) are the values of \( p' \) and \( q' \) and \( \mathcal{C}(x) \) is the ideal function to be approximated in S, we seek to

If \( c' \) is a vector of \( x \) and \( d' \) is a vector of \( x \) then the unknown coefficients to be de-

subject to

(4)

\[ 0 \leq (x)^d \]

\[ 0 \leq (x)^d + (x)^d \left[ \left( (x) + 1 \right) \right] \]

\[ 0 \leq (x)^d + (x)^d \left[ \left( (x) - 1 \right) \right] \]

with

After the kth iteration, an approximation to \( (c)^x \) is achieved, denoted by

for every \( x \in S \).

with the additional constraints

subject to

(3)

\[ \left\{ \left( (x)^d \right), \left( (x)^d \right) \right\} \]

(2)

\[ \left\{ \left( (x)^d \right), \left( (x)^d \right) \right\} \]

(1)

\[ \left( (x)^d \right) \left( (x)^d \right)^d = \left( (x)^d \right)^d \]

(0)
Moreover, at each point \( x \in S \), the variables \( \xi \) are defined through the

\[
E^x = C \frac{\partial}{\partial x} - (\frac{\partial}{\partial x})^x C
\]

as the objective criterion to be minimized:

At every sampling point \( x_i \in S \), let the following differences be considered:

\[\text{DETECTION OF THE ALGORITHM}\]

Digital filters are presented in the sequel to the approximation of single functions and the design of two-dimensional
response (the direct filter) of the multipliers of the approximated
harmonic applications such as the design of two-dimensional
information convergence to an optimum solution. These
incidental, environment convergence criteria to an optimum solution of
the overall computation time. It should be pointed out that the algorithm is easy
without requiring multiple iterations, thus effective a dramatic reduction of the
without requiring multiple iterations, thus effective a dramatic reduction of the
problem constraints \( \mathbb{N} \) variables and \( \mathbb{Z} \) constraints. It may be solved
problem constraints \( \mathbb{N} \) variables and \( \mathbb{Z} \) constraints. It may be solved
obtained directly by solving the dual problem only once. The resultant linear
obtained directly by solving the dual problem only once. The resultant linear
method based on a minimum formulation leading to the solution of a linear
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the shortcomings of the DCA approach may be overcome. The proposed
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problem of approximating real rational functions with LP methods, so that
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The main objective of this work is to provide a new formulation of the
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the chosen initial condition is to the final solution.
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In many cases not only on the form of the function itself but also on how close
dependent not only on the form of the function itself but also on how close
differential equation algorithm may be unstable, with the solution
the differential equation algorithm may be unstable, with the solution
but not obtainable. The function \( f \) and \( g \) have shown that
but not obtainable. The function \( f \) and \( g \) have shown that
the Lp part of the algorithm is
the Lp part of the algorithm is
\begin{align}
\left\| \frac{\partial}{\partial x} f(x) - (\frac{\partial}{\partial x})^x f(x) \right\| = 1
\end{align}

where

\[\text{Approximation of Real Rational Functions}\]
\[ g \leq \frac{\epsilon}{(\nabla x)\delta} \]

\[ \exists \delta \geq |(\nabla x)d - (\nabla x)\delta [\nabla x - (\nabla x)c]| \]

under the constraints

maximize \( g \)

The problem may be formulated as follows:

and the relations derived above are taken into consideration, the approximations

\[ \{ g \} \min g = \epsilon \]

must achieve a large positive value. An auxiliary variable \( g \) is defined by

\[ \frac{\epsilon}{(\nabla x)\delta} = g \]

then the quantity

\[ \| z \|^2 \max z = \epsilon \]

are minimized. Therefore,

\[ 0 < \epsilon \delta \text{ with } \frac{(\nabla x)\delta}{\| z \|^2} = \epsilon \delta \]

That the quantities that the quantities from (6) and (8), it is evident that the criterion of the form (g) implies

\[ \frac{(\nabla x)\delta}{\| z \|^2} = \frac{(\nabla x)d}{(\nabla x)d} = \nabla x - (\nabla x)c \]

The above relation may also be written in the form

\[ (\nabla x)d - (\nabla x)\delta [\nabla x - (\nabla x)c] = \| z \|^2 \]

Relation

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\[ 0 < \xi^2 \]

where \( A^x, S^x \in \mathbb{S} \).

\[ 0 \geq \xi + (I^x)^d N \sum b_{i-1} - \xi - \]

\[ (12) \quad 1 \geq (I^x)^d N \sum b_{i-1} + (I^x)^d N \sum \left[ E - (I^x)C \right] - \xi \left[ E - (I^x)C \right] - \]

\[ 1 \geq (I^x)^d N \sum - (I^x)^d N \sum \left[ E - (I^x)C \right] + \xi \left[ E - (I^x)C \right] \]

under the constraints.

maximize \( \xi \)

Now, using (16), the problem is formulated as

\[ \text{for } i = 1, \ldots, N \]

\[ \xi / b = b \\
\text{for } i = 1, \ldots, N \]

\[ \xi / d = d \\
\xi / i = \xi \]

may be converted to a linear one via the transformations.

The approximation problem, as formulated by (13), is not linear, but it is satisfied.

\[ \left. \left| \frac{(I^x)\hat{d}}{(I^x)} \right| - E - (I^x)C \right| \text{max } \xi \in \mathbb{R}^x = \xi \]

that the relation

solution of the problem (13), resulting in a minimum value for \( \xi \). Guarantees

absolute value to be less than or equal to \( \xi \). It is also obvious that the

and therefore the approximation error at each sampling point will be bounded

\[ \xi \geq \left| \frac{(I^x)\hat{d}}{(I^x)} \right| - (I^x)C \]

It is easily deduced from (13) that

Guarantees the positiveness of the denominator polynomial.

where \( \xi, \xi > 0 \). It is noted that the second set of constraints in (13)

Approximation of Real Rational Functions
The equality constraints result from the fact that the variables \( d_i \) and \( w_i \) are not signed.

\[
\begin{align*}
1 & \leq \cdots + c_n & + (z_n - 1) d_0 \\
0 & = \cdots + c_n 0 & + (z_n - 1) t_n (x)^d d - \\
& \vdots \\
0 & = \cdots + c_n 0 & + (z_n - 1) (x)^d d - \\
(18) & & \vdots \\
0 & = \cdots + c_n (x)^d d - (z_n - 1) (x)^d d (x) - (x) c] \\
& \vdots \\
0 & = \cdots + c_n (x)^d d - (z_n - 1) (x)^d d (x) - (x) c] \\
0 & \leq \cdots + c_n & - (z_n - 1) (x) c]
\end{align*}
\]

under the constraints

\[
\begin{align*}
0 & \leq 0 n + 0 n + 2 n + 2 n + 2 n + 1 n + 2 n + 1 n + 2 n + 1 n \\
\end{align*}
\]

The dual problem has the form

\[
\text{minimize } u n + w n + l n + 2 n + t n + z n + 1 n \text{ subject to the constraints...}
\]
Towards this goal, a total of eight approximations were derived for each function, corresponding to different values for $N$ and $M$. The sample population, with a stepsize of 0.05. For each sampling point, $E_k = 0.5$, Table 1 summarizes the complete results for this example, while Figures 1 and 2 depict the approximations to $G(x)$ as well as the errors for $G(x)$ they are in the interval $[0.1, 1.1]$ with a stepsize of 0.1, whereas for $G_2(x)$ the sampling points lie in the interval $[1, 1.1]$. It is finally noted that the approximation method for these examples varies from six to twelve, as compared with a single iteration needed with the proposed approach.

In this example, the results are presented from the proposed approach and the DCA method. The DCA results are abstracted from Reference [4] and refer to the approximation of the functions $G(x) = e_x$ and $G_2(x) = e_x$ with a rational function of the form $H(x) = \frac{\sum_{i=0}^{N} a_i x^i}{\sum_{i=0}^{N} b_i x^i}$.

$$H(x) = \frac{\sum_{i=0}^{N} a_i x^i}{\sum_{i=0}^{N} b_i x^i}.$$  

(19)

The dimensions of the coefficient array $E(k)$ are $(N + M + 2 \times 3K)$ symmetric and sparsity characteristics, but primarily the specific functional form of this array, keep storage requirements to a minimum. As a matter of fact, since the expressions $Q(x), P(x), Q_2(x)$, as well as those for $G(x)$ may be determined algorithmically without any further need to store its value. The importance of this assertion may be illustrated with an example. If $E_k \in S$ and $E_k \notin S_2$, the RSA method itself is accelerated because of the form of the coefficient array. Furthermore, the $E_k = 1$, for each sampling point, only one iteration is required instead of three. For the revised simplex algorithm, it is finally noted that additional desirable constraints may be integrated into the linear problem (17) by dividing both sides of the expressions by $\xi$ and applying subsequently the transformations (16).
\[
\frac{\left(\frac{z^-1}{z^2}\right)D \left(\frac{1}{z^2}\right)H}{\left(\frac{z^-1}{z^2}\right)D \left(\frac{1}{z^2}\right)} = \left(\frac{z^-1}{z^2}\right)D = \frac{z^2}{H}
\]

The MRI filter may be written as

\[\text{Example 2}
\]

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<th>(2)</th>
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<tr>
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<tr>
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<td>4.0</td>
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<tr>
<td>2.0</td>
<td>3.0</td>
<td>1.0</td>
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<td>2.0</td>
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</tbody>
</table>

Approximation results for Example 1

<table>
<thead>
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<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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</thead>
<tbody>
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<td>3.0</td>
<td>4.0</td>
<td>5.0</td>
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<td>4.0</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Table 1

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Approximation of Real Rational Functions
which may be grouped now with the main constraints of (17):

\[(42) \quad \{z_1 z + \gamma z \} = \gamma m\]

Since, for this case, \(P_{70} \neq 0\), this constraint must be integrated into the linear problem formulation (17). Towards this end, division by \(z \gamma m\) and application of the transformations (16) lead to the linear constraints

\[\begin{align*}
\gamma z_1 m &\geq \gamma m \\
\gamma z_0 m &\geq \gamma z_0 m \\
\gamma z_0 m &\geq 0
\end{align*}\]

For the ideal function \(C = (\gamma z_0 m, \gamma z_0 m)\), to be approximated is specified as

\[1 + z_1 s + s = \gamma m \quad \text{and} \quad z_1 z + \gamma = \gamma S \quad 1 + z_1 z = \gamma S\]

where

\[\begin{align*}
(\gamma z_0 m) \quad (\gamma z_0 m)
\end{align*}\]

for

\[\begin{align*}
\gamma z_0 m &\geq \gamma m \\
\gamma z_0 m &\geq \gamma z_0 m \\
\gamma z_0 m &\geq 0
\end{align*}\]

The solution to (42) is

\[1 + \frac{\gamma z_1 m}{\gamma z_0 m} = \gamma m \quad \text{and} \quad \gamma z_1 m + \gamma = \gamma S \quad \gamma z_1 m + \gamma = \gamma S\]

where

\[\begin{align*}
(\gamma z_0 m) \quad (\gamma z_0 m)
\end{align*}\]

Moreover, all their coefficients equal to unity. For the example under investigation, the sampling point \(z \gamma m\) is the normalized frequencies at the places of the polynomials of \((\gamma z_1 m)\) and \((\gamma z_1 m)\) \(d \gamma m\) with the transformed \(z \gamma m\) with the normalized frequencies at the places where

\[\begin{align*}
\frac{\gamma z_1 m}{\gamma z_0 m} = \gamma z_1 m \quad \text{and} \quad \gamma z_1 m = \gamma S
\end{align*}\]
Approximation of Radial Radial Functions
REFERENCES

ness of the algorithm in attacking real design problems.

The two examples presented clearly illustrate the usefulness and effective-
ness of convolutional methods becomes problematic.
The dimensionality of the approximation problem is large and the application
of convolution is desirable through a revised simplex algorithm using the
plot of the error. Figure 4 shows a graphically representation of the WFTL.

Figure 3 shows a graphically representation of the WFTL. The solution
presented here is achieved via a single iteration only.

It is important to note that if the same design problem is approached using

\[ \sum_{i=0}^{N} \left( \sum_{j=0}^{M} w_{ij} y_{ij} \right) = 1 - \gamma \]

while the values for \( \gamma \) and \( \delta \) are found to be \( \gamma = 0.03362449 \), \( \delta = 0.05890589 \),

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